Project 3 | Mission: Improbable

Summary Report

Alan Brantley

Description and Reasoning

Characterizing Motor and Gearbox

To characterize the motor and gearbox, I conducted three experiments from which I would use to calculate my values. I performed a no-load test, a stall test, and a speed test. Each test was performed at various voltages in .5 Volt intervals and allowed for several seconds between steps.

I calculated R first using data from the stall test. To calculate k, I first found the angular velocity. For this experiment, I used the single-arm servo horn and recorded the motor in slow motion at various speeds. Using these data, I calculated k for 3 <= **V** <= 6 and took an average.

Given **k**, I calculated $\mathbf{T}_{fric}$, which in turn allowed me to calculate Tload at various voltages.

Choosing Values

I choose the values of **V**, $\mathbf{R}_g$, and $\mathbf{C}_r$ through a process of iterative experimentation. I tested multiple combinations of gear ratios and capstan radii. In total, I performed lift tests with Gear ratios of 80:1, 25:1, and 20:1 with capstan radii of 7mm, 11mm, 22mm, 24.5mm, and 27.5mm.

I chose $\mathbf{V}_{app}$ **= 6V** because I wanted to lift the payload as fast as possible. In my experiments, the speed with which the payload lifted was directly proportional to the voltage.

I successively tested 80:1, 25:1 with different radii. Each step down resulted in faster times. Finally, I tested at 20:1 with 11mm and 7mm radii.

My fastest time was achieved with a combination of $\mathbf{r}_c$ = 11mm with a gear ratio of 20:1.

Estimates:

$R_c = 0.8135\Omega$

$k = 0.002797 \cdot \frac{Nm}{A}$

$T_{fric} = 0.000770\ N \cdot m$

$\eta_{stage} \approx .509$

Chosen:

$V_{app} = 6V$

$R_g = 20 : 1$

$r_{cap} = 0.011m$

Experimental:

$t_{lift} = 1.59s$

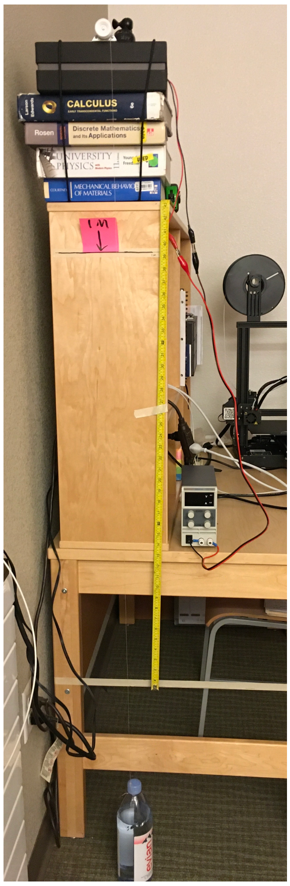
$i_{mot} = 4.40A$

$P_{elec} = 26.39W$

$\eta_{sys} \approx .21$

$\eta_{mot} \approx .19$

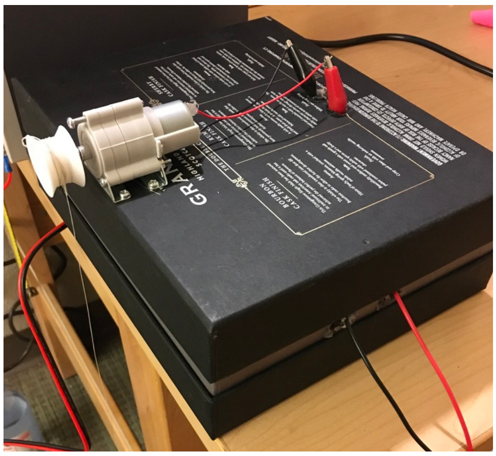
$\eta_{trans} \approx .398$



Lift Test Setup

Power and Efficiency

I calculated system power and efficiencies using data collected in my lift experiment and calculated values. During my lift test, I measured current and voltage to find $\mathbf{P}_{elec}$. I calculated system efficiency by taking the ratio of $\mathbf{P}_{out}$ = $\mathbf{Fd/t}_{lift}$ with $\mathbf{P}_{elec}$. Next, I calculated $\mathbf{P}_{mech}$ over $\mathbf{P}_{elec}$ for motor efficiency. Finally, I calculated transmission efficiency by taking the ratio of the power delivered to the capstan and the power from the motor.


Motor Mount

Motor and Gearbox Characterization

Table 1 — Measured / Calculated

V_{app}	i_{nl}	i_{stall}	$i_{@6V}$	rev/s	R_c	$\dot{\theta}_m$	$\dot{\theta}_{nl}$	k	T_{fric}
	no load test	stall test	speed test | 25:1						
1	.18	1.24			0.806				
1.5	.19	1.70			0.882				
2	.20	2.40			0.833				
2.5	.21	3.60			0.694				
3	.22	3.47	1.32	6	0.864	37.70	942.50	0.00307	0.000615
3.5	.24	4.20	1.33	8.5	0.830	53.41	1335.50	0.00254	0.000671
4	.27	4.60	1.37	9.5	0.870	59.69	1492.25	0.00261	0.000755
4.5	.31	5.36	1.38	10	0.840	62.83	1570.75	0.00280	0.000867
5	.25	7.00	1.39	11.5	0.714	72.26	1806.50	0.00271	0.000699
5.5	.27	7.36	1.43	12	0.747	75.40	1885.00	0.00287	0.000755
6	.28	6.90	1.45	12.5	0.869	78.54	1936.50	0.00300	0.000783

Resistance, R_c:

$$i_{Stall} = \frac{1}{R} V_{app}$$

$$R_c = \frac{1}{i_{stall}} V_{app} = 0.8135\Omega$$

Average of R_c values calculated in **table 1** with data from the **stall test**.

Motor constant, k:

$$T_{em} = k \cdot \dot{\theta}_m$$

$$V_{app} = V_{em} + iR$$

$$k = \frac{V_{app} - iR}{\dot{\theta}_m} = 0.002797 N \cdot m$$

Average of **k** values calculated in **table 1** with data gathered from the **speed test**. Note that while the speed test was conducted at 25:1, my final test setup used a 20:1 gear ratio.

Motor friction torque, τ_{fric}

$$T_{em} = ki$$

$$T_{load} = T_{em} - T_{fric}$$

When $T_{load} = 0$,

$$T_{fric} = T_{em} = k \cdot i_{nl} = 0.000770 \; N \cdot m$$

Average of T_{fric} values calculated in **table 1** using **k** and data obtained during the **no load test**.

Per-stage efficiency, η_{stage}

$$T_{in} \cdot GR \cdot \eta = T_{out}$$

$$\eta_{stage} = \sqrt{\eta_{gearbox}} = \sqrt{\frac{T_{out}}{T_{in} \cdot GR}} = \sqrt{\frac{mgr_{capstan}}{T_{load} \cdot GR}}$$

$$= \sqrt{\frac{8.96N \cdot 0.011m}{0.0190Nm \cdot 20}} \approx .509$$

Where the gear ratio is 20:1, $\mathbf{r_c}$ = 0.011 m, and $\mathbf{T_{load}}$ is measured at $\mathbf{V_{app}}$ = 6V (table 2). The square root comes from the fact that there are two stages.

Table 2

V_{app}	$\dot{\theta}_{m\text{-}20:1}$ (rad/s)	T_{load} (Nm)
3	47.125	0.00909
3.5	66.775	0.0106
4	74.613	0.0122
4.5	78.538	0.0139
5	90.325	0.0155
5.5	94.250	0.0172
6	96.825	0.0190

Power and Efficiency Calculations

Electrical power, P_{elec}

$$P_{elec} = iV_{app-lift} = 4.40A(6V) = 26.39W$$

These values were measured during the lift test. ***i*** was calculated using the average of the readable currents during lifting.

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System efficiency, η_{sys}:

$$\eta_{sys} = \frac{P_{out}}{P_{in}} = \frac{P_{GB}}{P_{elec}} = \frac{5.595W}{26.39W} \approx 0.21$$

Where P_{GB} **= Fd/t_{lift}** and P_{elec} is calculated above.

Motor efficiency, η_{motor}:

$$P_{mech} = (\frac{k}{R} \cdot V_{app} - T_{fric})\dot{\theta}_m - \frac{k^2}{R}\dot{\theta}_m^2$$

$$= 4.89W$$

$$\eta_{motor} = \frac{P_{mech}}{P_{elec}} = \frac{4.89W}{26.39W} \approx 0.19$$

Where P_{mech} is calculated at 6V and the velocity is calculated from data gathered during the speed test (table 2) and converted with 20:1 GR.

Transmission efficiency, η_{trans}:

$$\eta_{trans} = \frac{T_{out} \cdot \dot{\theta}_{out}}{T_{motor} \cdot \dot{\theta}_{motor}} = \frac{F \cdot r_c \cdot \dot{\theta}_{out}}{k \cdot i \cdot \dot{\theta}_{motor}} = \frac{8.896N \cdot 0.011m \cdot 96.825\,\frac{rad}{s}}{0.002797\,\frac{Nm}{A} \cdot 4.4A \cdot 96.825\,\frac{rad}{s} \cdot 20} \approx 0.398$$